MEASUREMENT OF AGGREGATE OUTPUT: CASE FOR A MOVING WEIGHT INDEX FOR INDIA

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Any study that deals with the aggregate of a number of heterogeneous products is faced with the problem of how to add these products up. Essentially the problem is one of choosing the appropriate weights to combine the dissimilar items. The problem becomes complicated when comparisons over space and time are involved, and prices, the natural choice of a weight in such cases, vary over observations.

The most common measures of aggregate output rely on fixed price weights using either initial year prices (Laspeyres' Index) or final year prices (Paasches' Index). Apart from neglecting relevant information on the time variation in prices when such information is available, these fixed weight indices are based on the assumption that relative prices of different commodities are constant between regions and years. In reality this assumption is seldom valid. The same total physical output configuration, valued at different relative prices, will show apparently different measures of output levels. To minimize this problem weights have sometime [1] been defined as the geometric means of the initial and final year prices, where it is assumed that the downward and upward biases are offsetting. In addition to still being wasteful of price information, this measure reflects neither the initial nor the final year weight, and in reality the biases may not be offsetting. Moreover, the problem of inter-regional comparison still remains.

The problem posed by the assumption of constant relative prices can be dealt with in an output measure that uses weights changing freely between regions and years. Such a measure is offered by the

Divisia Index which has been proved to possess the unique property of a minimum error of approximation as the economy moves from one production configuration to another [5]. Following Jorgenson and Griliches [2], [3] an argument for Divisia Index is presented below.

The basic objective of the Divisia measure of output is to approximate a quantity measure as distinct from a value measure. This is done by separating the index of value products into price and quantity indexes. Let v, p, q represent value, price and quantity respectively, and $\dot{v}, \dot{p}, \dot{q}$ the corresponding time derivatives.

For n commodities,

$$v = v_1 + v_2 + \ldots + v_n$$

$$= p_1 q_1 + p_2 q_2 + \ldots + p_n q_n$$

Differentiating with respect to time we have :

$$\frac{\dot{v}}{v} = \left[\frac{v_1}{v}\frac{\dot{p}_i}{p_i} + \frac{v_2}{v}\frac{\dot{p}_2}{p_2} + \ldots + \frac{v_n}{v}\frac{\dot{p}_n}{p_n}\right] +$$

$$\left[\frac{v_1}{v}\frac{\dot{q}_1}{q_1} + \frac{v_2}{v}\frac{\dot{q}_2}{q_2} + \ldots\ldots + \frac{v_n}{v}\frac{\dot{q}_n}{q_n}\right]$$

$$\frac{\dot{v}}{v} = \sum_i w_i \frac{\dot{p}_i}{p_i} + \sum_i w_i \frac{\dot{q}_i}{q_i} \qquad \ldots(I)$$

where $w_i = \dfrac{v_i}{v} = \dfrac{v_i}{\sum_i v_i} = \dfrac{p_i q_i}{\sum_i p_i q_i}$

the left hand side of expression I denotes the rate of change of the value of output, the first term on the right hand side denotes the weighted rate of change of prices, and the second term on the right hand side denotes the weighted rate of change of quantity, the weights being the share of each commodity in the total value of output. Thus the rates of growth of the Divisia price and quantity measures add up to the rate of growth of the Divisia value measure. This yields an index of quantity of output aggregated by using value shares as weights that can change continuously. For application to discrete points of time (years) this index can be modified as follows [1], [6] :

$$\left[\log q_t - \log q_{t-1}\right] = \sum_i \bar{w}_{it}\left[\log q_{it} - \log q_{i,t-1}\right] \qquad \ldots(II)$$

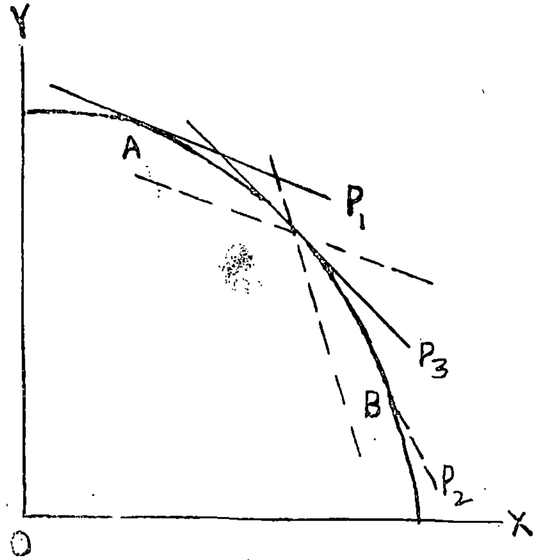
where $\bar{w}_{it}$ is the arithmetic mean of the weights (relative shares of commodities in the total value) in two years t and $t-1$, i.e.,

$$\bar{w}_{it} = \frac{w_{it} + w_{i,t-1}}{2}$$

The measure of output presented above provides an efficient approximation of the quantity in the sense of (*i*) avoiding the problem of changing relative prices, (*ii*) satisfying the time-reversal test, and (*iii*) satisfying the factor reversal test [7].

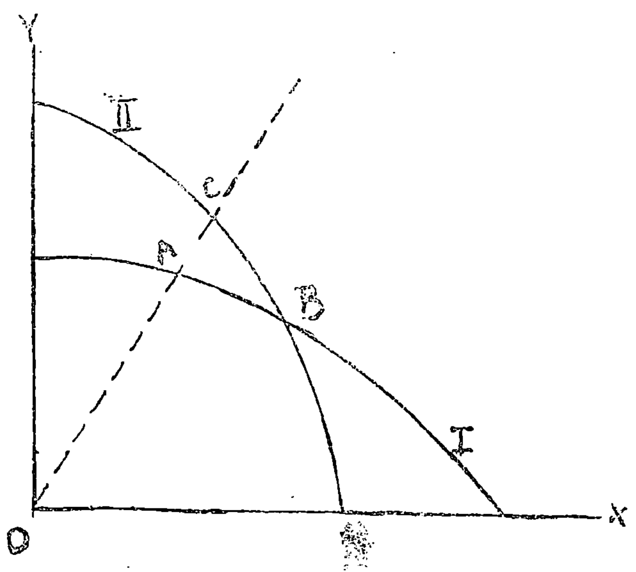
It can be shown that when relative prices do not remain constant, and time-series and cross-section observations are pooled for economic analysis, Laspeyres' and Divisia measures would make it possible to explain different proportions of the variations in the aggregate measure of output [4].

In figure I below let A and B represent the output compositions and p_1 and p_2 the relative prices in region 1 and 2 respectively in year 1. Now suppose over time relative prices have changed in both the regions with region 1 moving to produce more of X and region 2 to produce more of Y so that in year 2 both produce the output configuration C at the same relative price p_3.



Now if Divisia measure is used, the configuration C will represent the same aggregate output for both regions, whereas if Laspeyres' measure is used, C will be evaluated at the two relative prices p_1 and p_2 and will yield two aggregate output measures for the two regions. Thus if different regions have systematically changing relative prices over time so that there is a tendency for them to converge, an econometric analysis using output as the dependent variable might indicate that a smaller proportion of variation is explained in the Laspeyres' measure than in the Divisia measure.

However, the validity of the Divisia Index is based upon one important assumption, that is, the index at time t is not independent of the path by which the output level at time t was attained [5]. This is illustrated in figure 2.



Consider a region with two goods X and Y with a given endowment of inputs. I describes the product transformation curve for the region, and in the initial period the output configuration is given by A. Now suppose the economy of the region slides down on the product transformation curve, so that the output configuration becomes B.

Since both A and B are on the same product transformation curve, the Divisia Index of output will remain unchanged. Now suppose there is a change in the input composition in the region so that it is now technically possible to produce new configurations of the two products with the product combination B being still available. (Example of such a situation may be the simultaneous occurrence of a plant disease for one crop and an improved seed for the other) In other words, there is a new product transformation curve (denoted by II in the figure) with the point B being common to both the old and new product transformation curves. During the shift to the new product transformation curve the output index remains unchanged because there has not yet been any change in the output configuration B. Now suppose there is again a rising demand for Y and a rise in its relative price so that there is a movement along the new transformation curve to C. The invariance axiom of Divisia Index guarantees that the output index will remain the same at C as at B.

But now consider a movement direct from A to C without taking into account the actual path travelled by output. In this case, since C is to the north-east of A, there will be an increase in the output index. Thus Divisia Index remains unchanged if we consider the path travelled by the output but increases if we ignore the path. Divisia Index does assume that the index of the final configuration (C) is not independent of the path travelled (from A to C via B) by it, and the path determines the change (or lack of it) in the output index. The applicability of Divisia Index, therefore, depends on the main purpose of the analysis involved. If the purpose implies assumption of independence of the time path taken by the output, and only terminal points are considered, Divisia Index is not the answer. Otherwise, this will provide a measure free from the biases of the indexes. In an econometric analysis involving time-series and cross-section data Divisia Index does seem reasonable, especially if the time and cross-section units are not widely disparate.

The author has calculated Divisia and Laspeyres Indexes for aggregate crop output for 72 districts in the predominantly wheat producing areas of India for the period 1959-60 to 1968-69.* The indexes for five districts selected from five states are presented for illustration in the Appendix. It shows that the two indexes are fairly close to each other for the initial years, but in general they diverge over time and suggest quite different rates of change over the period, indicative of the effect of changing relative prices. Conclusions regarding measured rates of growth of income or aggregate output at

*These are aggregates of 30 crops produced in these districts. The data were made available to the author by the Economic Development Center, University of Minnesota.

national or regional levels would, therefore, vary depending on which index is used, and if the purpose is to minimise the biases arising out of changing relative prices, the Divisia Index should be used (subject, of course, to the invariance axiom). A production function analysis to explain variations in aggregate output has been carried out by the author using both sets of indexes as dependent variables. From a comparison of the two it appears that the effects of the recent technological changes in crop production and its probable consequences on output composition seem to be reflected more in the results based upon the Divisia Indexes[4].

SUMMARY

This note has sought to indicate that measures of aggregate national or regional output, their rates of change and results of analyses based on these aggregates are critically-dependent upon the choice of the index. Under certain conditions, the Divisia Index offers a better approximation to the quantity measures of output and would, therefore, be more dependable for economic policy. For regional or temporal comparisons and for evaluating the outcomes of investment programmes, Divisia Index seems preferable. The availability of a series of this index along with the traditional fixed weight indexes would be of much use to analysis and policy makers.

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APPENDIX

Index of the aggregate crop output in Selected districts
(base 1959-60 = 100)

Year	DISTRICT							
	Hoshangabad [*M.P.*]		*Shivpuri* [*M P.*]		*Jullundhur* [*Punjab*]		*Karpurthala* [*Punjab*]	
	A	*B*	*A*	*B*	*A*	*B*	*A*	*B*
1	2	3	4	5	6	7	8	9
1959-60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1960-61	112.46	124.88	113.27	113.15	99.27	99.26	99.45	99.15
1961-62	80.97	89.73	98.75	97.84	101.87	101.38	100.13	101.45
1962-63	107 08	136.31	115 33	125.37	105.44	105.41	91.88	92.11
1963-64	112.39	148.02	128.43	141.26	106.18	105.27	112.09	115.05
1964-65	142.60	190.08	117.54	131.10	134.59	133.37	123.62	126.98
1965-66	136.00	184.31	91.44	102.32	118.19	129.39	109.42	177.29
1966-67	112.76	150.79	76.55	87.98	107.18	121.68	129.72	261.53
1967-68	125.21	169.11	122.47	143.20	116.05	131 82	131.80	274 12
1968-69	133.70	182.17	97.91	116.41	133.74	150.45	161.06	332 35

APPENDIX (*Contd.*)

	Ganganagar (*Raj.*)		*Mathura* (*U.P.*)		*Gonda* (*U.P.*)		*Gurgaon* (*Haryana*)	
	A	*B*	*A*	*B*	*A*	*B*	*A*	*B*
1	10	11	12	13	14	15	16	17
1959-60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1960-61	96.32	93.61	72.97	72.23	104.17	103.23	124.71	124.98
1961-62	102.36	103.67	60.48	59.71	124.36	126.68	145.58	148.92
1962-63	100.65	104.05	97.09	102.52	106.56	110.16	141.53	145.49
1963-64	97.49	92.02	62.25	73.48	101.88	103.61	111.85	116.13
1964-65	111.74	119.29	73.92	88.62	125.95	132.26	138.68	149.54
1965-66	85.45	85.63	102.46	122.74	117.29	125.82	135.59	149.64
1966-67	112.91	136.88	89.75	106.32	85.64	90.56	155.46	171.33
1967-68	166.42	200.83	73.01	91.36	136.89	152.94	217.74	244.86
1968-69	105.38	122.54	86.60	108.30	147.72	170.52	181.61	203.88

A : Laspeyre index

B : Divisia index